Uncertainty

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Applied Biomathematics

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Kinds of uncertainty

- Variability
  - aleatory uncertainty, stochasticity, randomness, Type A
- Incertitude
  - epistemic uncertainty, imprecision, uncertainty, Type B
- Vagueness
  - semantic uncertainty, fuzziness, multivalent uncertainty
- Confusion, etc.

Incertitude

- Arises from incomplete knowledge
- Incertitude arises from
  - limited sample size
  - mensurational limits (‘measurement error’)
  - use of surrogate data
- Reducible with empirical effort

Variability

- Arises from natural stochasticity

- Variability arises from
  - spatial variation
  - temporal fluctuations
  - genetic or manufacturing differences

- Not reducible by empirical effort

Two kinds of uncertainty

- Variability
- Aleatory uncertainty
- Type A uncertainty
- Stochasticity
- Randomness
- Chance
- Risk

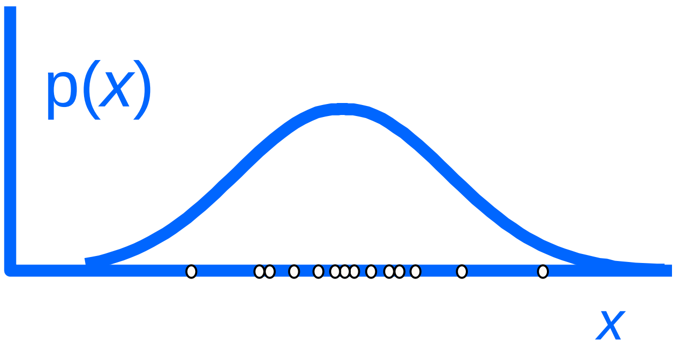
- Incertitude
- Epistemic uncertainty
- Type B uncertainty
- Ambiguity
- Ignorance
- Imprecision
- True uncertainty

Propagating variability

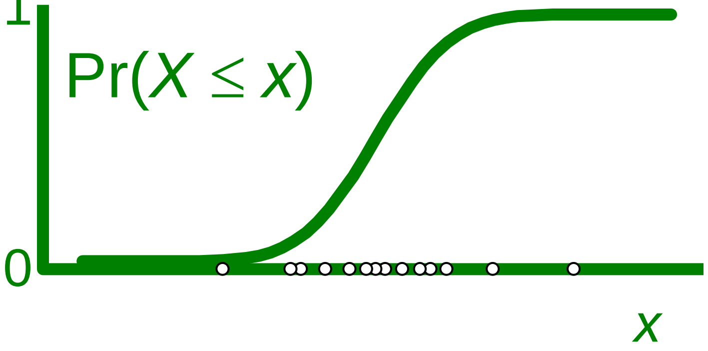
- Probability theory projects variability through mathematical models
- Suppose
    - Doses vary among individuals
    - Susceptibilities also vary among individuals
- Model both by probability distributions

Distribution of random number X

Density distribution



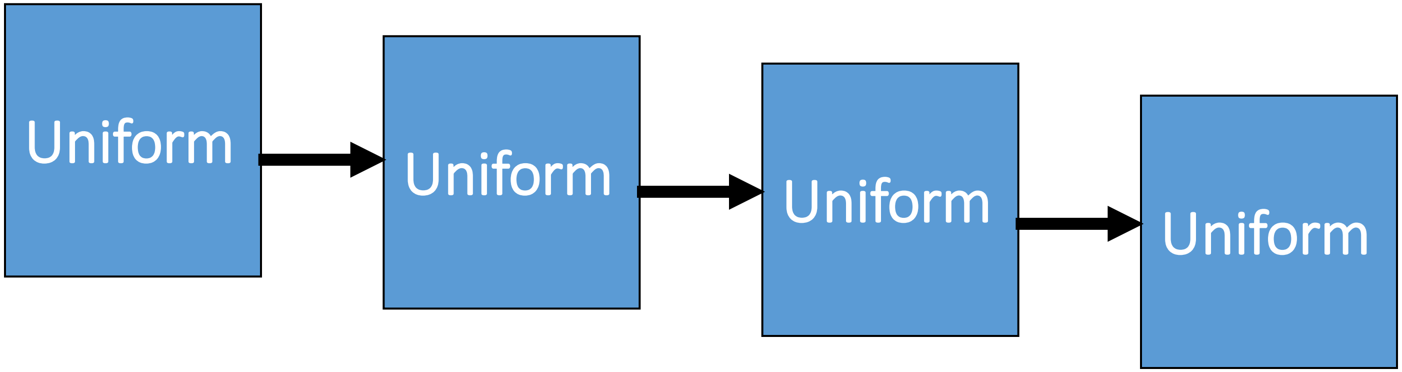
Cumulative distribution

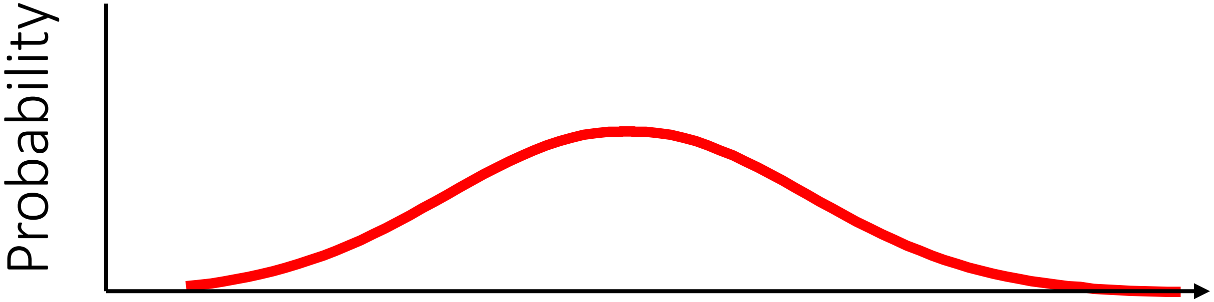


Accumulating gross uncertainty

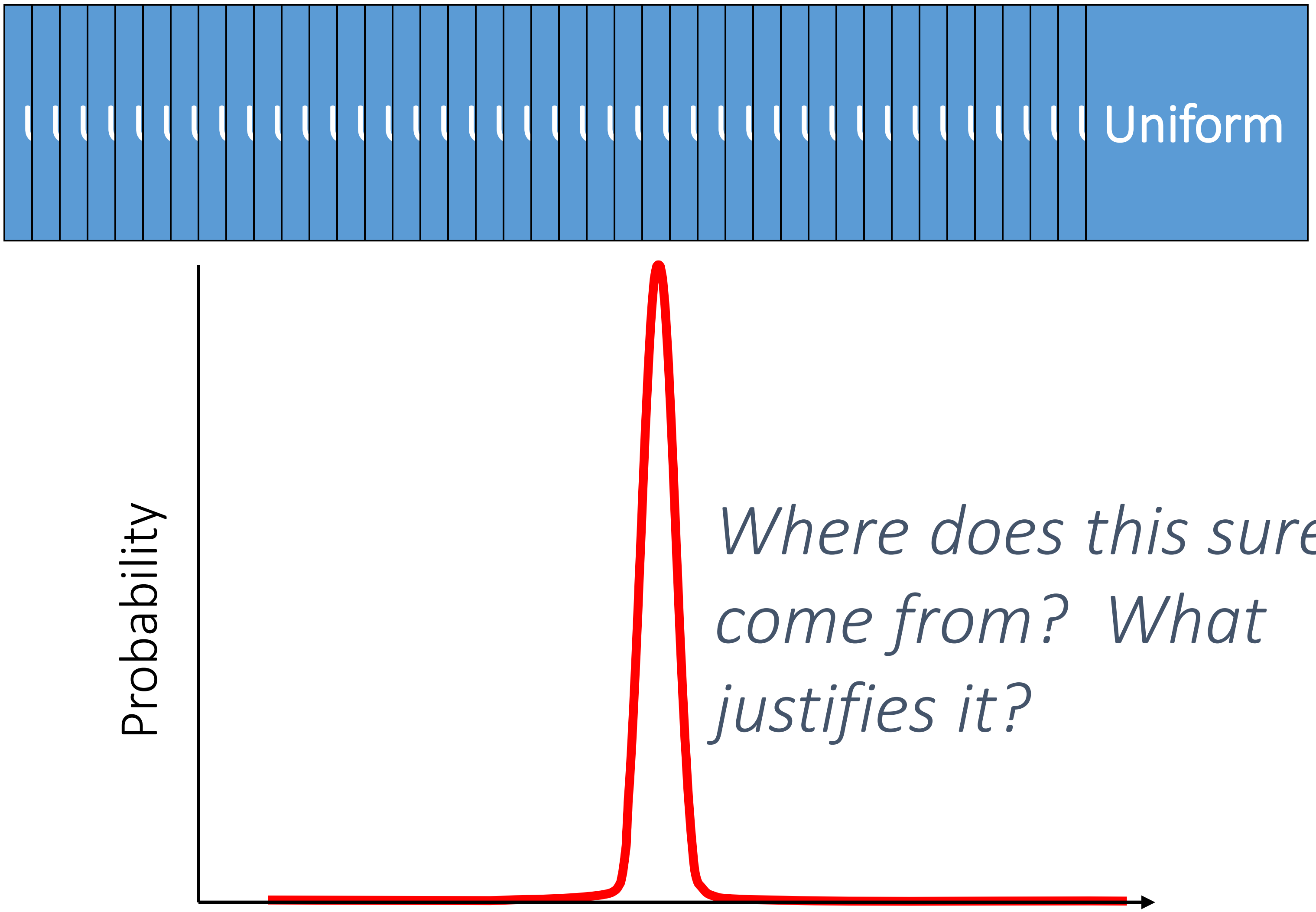
- Probability doesn’t do it in an intuitive way
- Output precision of the answer depends on the number of inputs
- The more inputs, the tighter the answer

A few grossly uncertain inputs





Many grossly uncertain inputs...



Where does this surety come from? What justifies it?

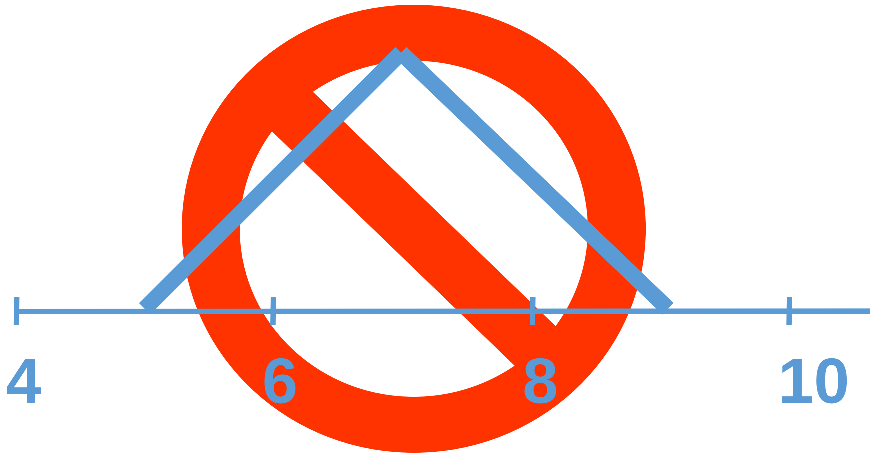
Propagating incertitude

Suppose

A is in [2, 4]

B is in [3, 5]

What can be said about the sum *A*+*B*?



The right answer is [5,9]

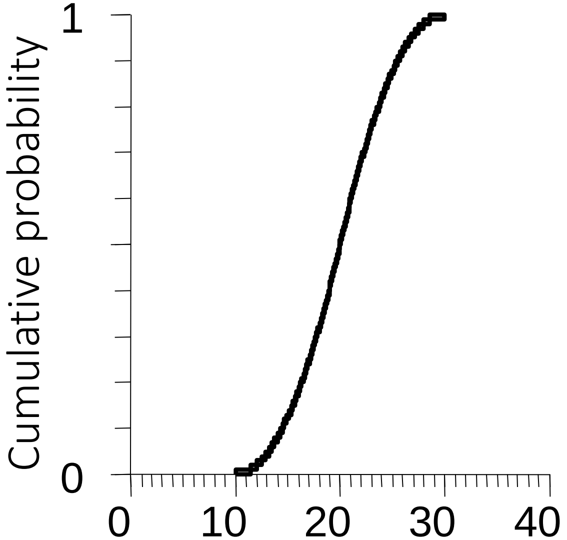
Don't use probability!

They must be treated *differently*

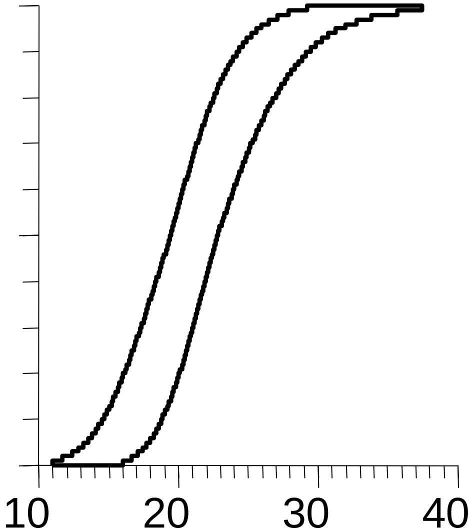
- Variability = randomness $\Rightarrow$ probability theory
- Incertitude = ignorance $\Rightarrow$ interval analysis
- Imprecise probabilities can do both at once

Uncertain numbers

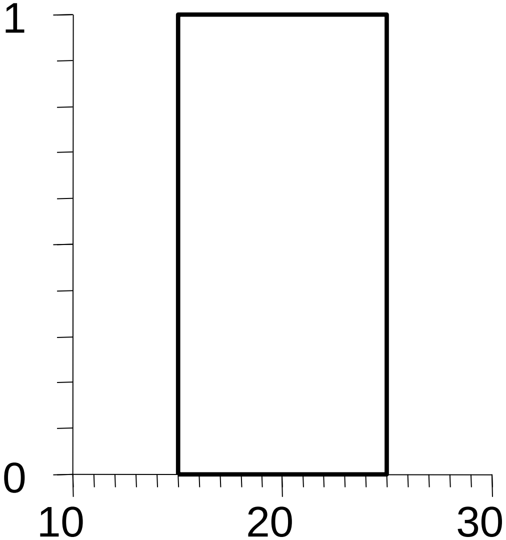
Probability distribution



Probability box (p-box)



Interval

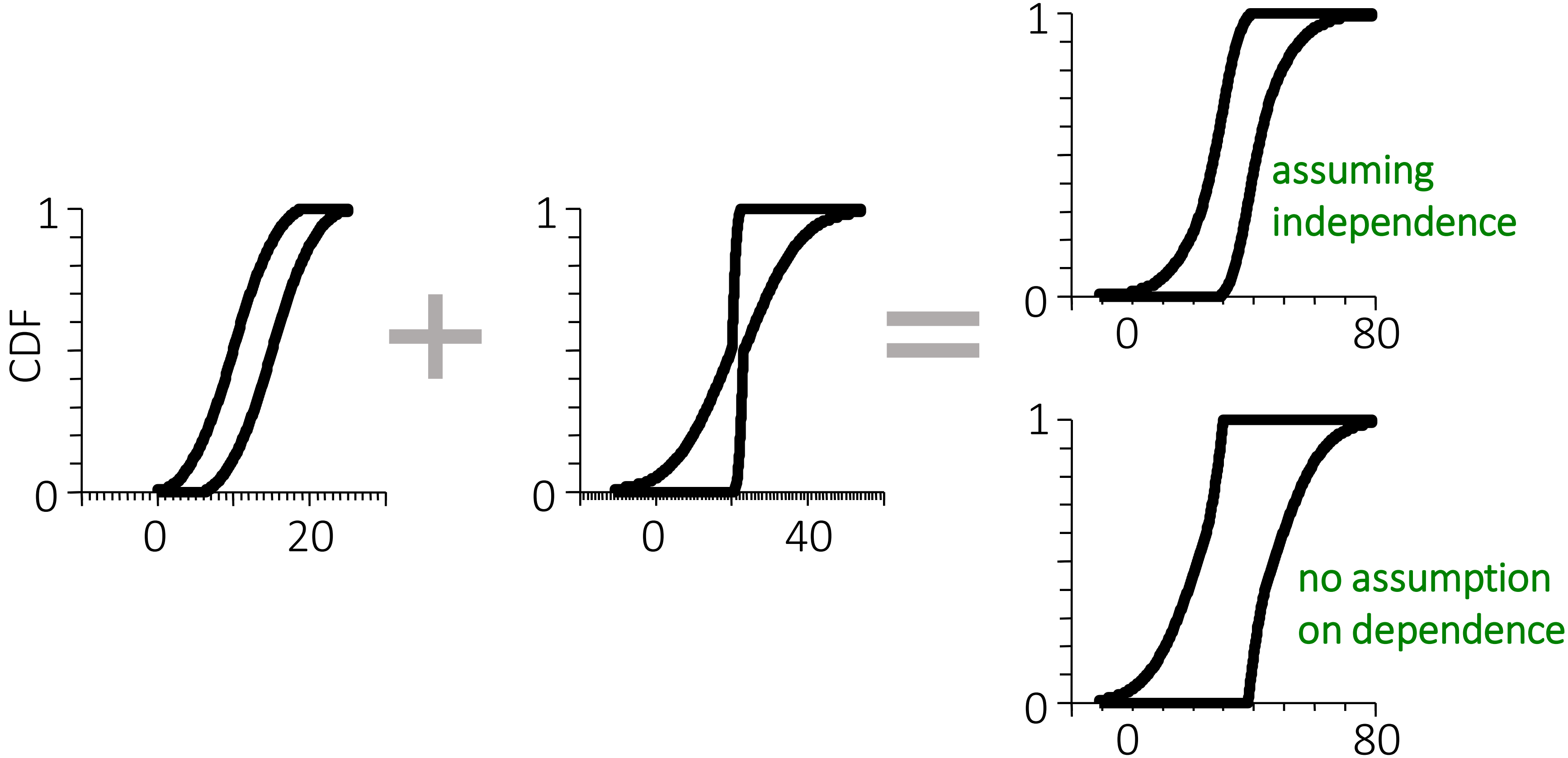


Not a uniform distribution

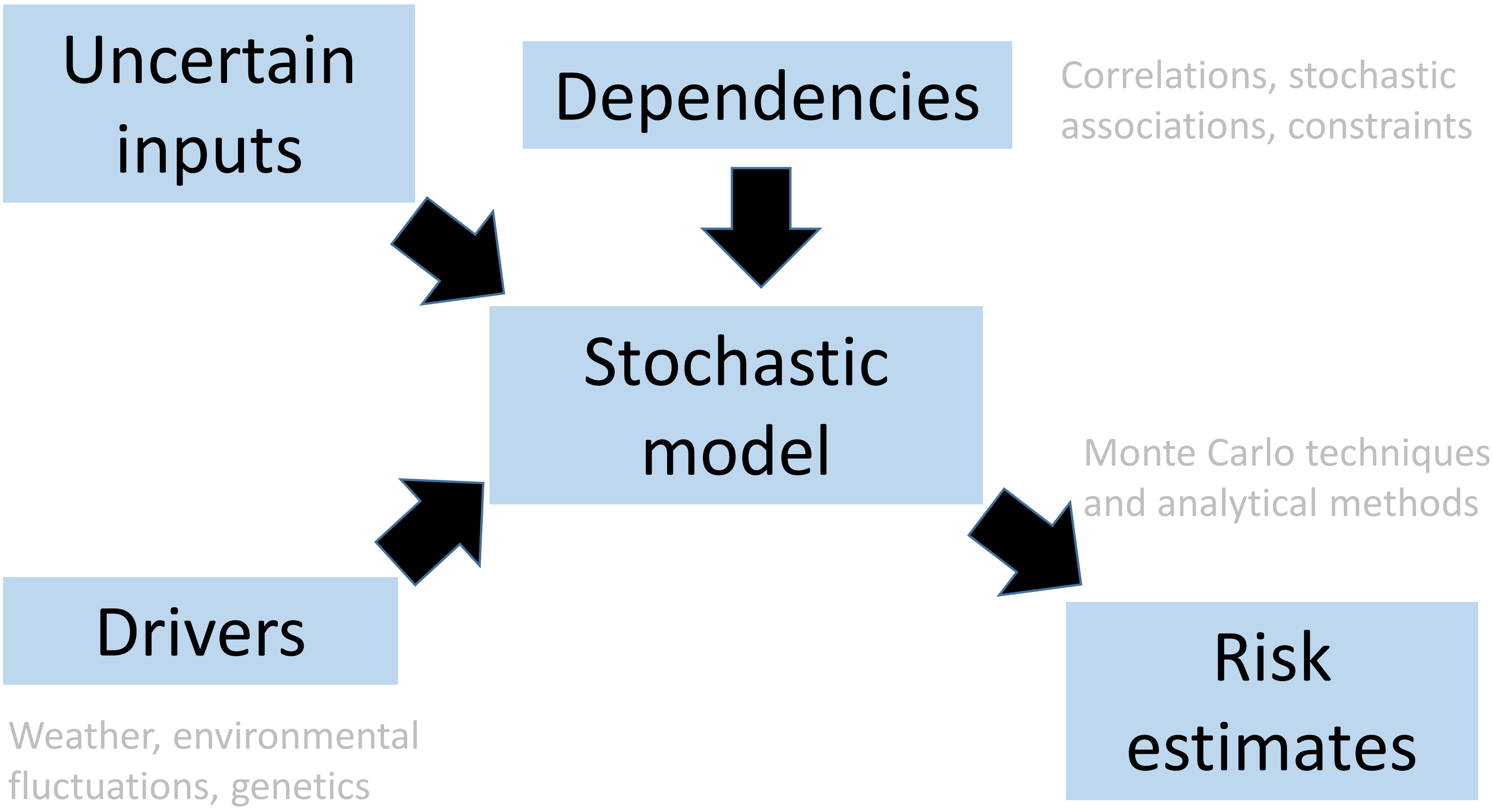
Uncertainty arithmetic

- We can do math on imprecise probabilities
- When inputs are distributions, agrees with probability theory
- When inputs are intervals, agrees with interval analysis

Example


CDF
1
0
0
20
+
1
0
0
40
=
1
0
0
80
assuming
independence
1
0
0
80
no assumption
on dependence

Uncertainty propagation

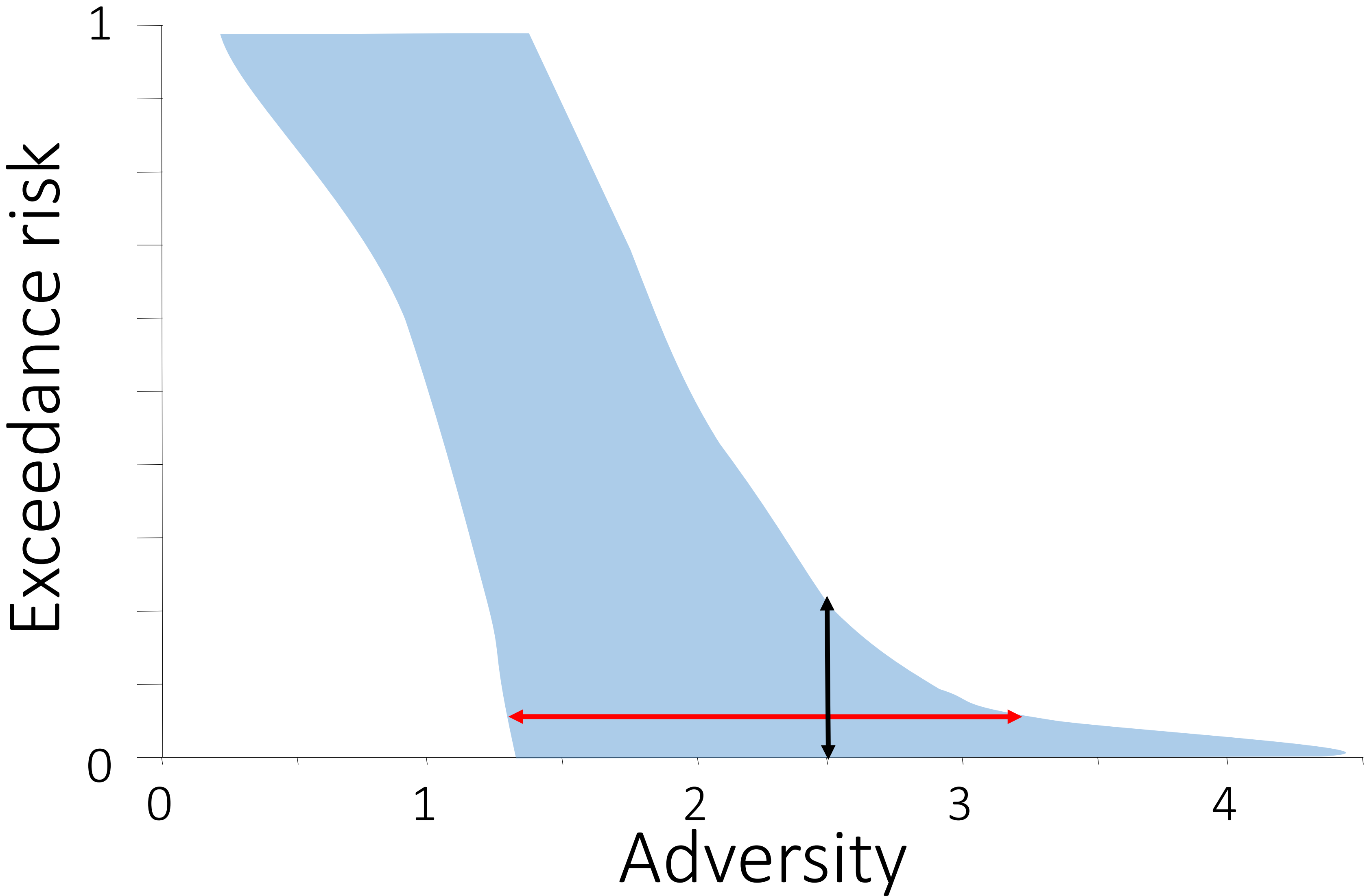


Sources of incertitude

- Doubt about the model or biological mechanism
- Uncertainty about the shape of distributions
- Uncertainty about the dependence
- Measurement imprecision in data
- Incomplete or missing data
- Residual fitting error

Dependence can be complex

Typical results



How to use the results

When uncertainty makes no difference

(because results are so clear), bounding gives confidence in the reliability of the decision

When uncertainty obscures the decision

(*i*) use results to identify inputs to study better, or

(*ii*) use other criteria within probability bounds

Can uncertainty swamp the answer?

- Sure, if uncertainty is huge
- This *should* happen (it's not "unhelpful")
- If you think the bounds are too wide, then put in whatever information is missing
- If there isn't any such information, do you want to *mislead* people?

Probability vs. bounding

- Interval (worst case) analysis
  - Good for incertitude (epistemic uncertainty)
  - Bad for frequency and dependence
  - Saying *less* than you know (cowardice)

- Probability theory
  - Good for likelihoods and dependence
  - Bad for incertitude
  - Saying *more* than you know (misleading)

Everyone makes assumptions

- But not all sets of assumptions are equal!

Point value	Linear function
Interval range	Monotonic function
Entire real line	Any function
Normal distribution	Independence
Unimodal distribution	Known correlation
Any distribution	Any dependence

- Like to discharge unwarranted assumptions

“Certainties lead to doubt; doubts lead to certainty”

End